Published by Institute of Physics Publishing for SISSA/ISAS

Received: *May 02, 2002*
Accepted: *May 08, 2002*

Open strings in exactly solvable model of curved spacetime and pp-wave limit

Hiromitsu Takayanagi and Tadashi Takayanagi

Department of Physics, Faculty of Science, University of Tokyo
Hongo 7-3-1, Bunkyo-ku, Tokyo, 113-0033, Japan
E-mail: hiro@hep-th.phys.s.u-tokyo.ac.jp,
takayana@hep-th.phys.s.u-tokyo.ac.jp

Abstract: In this paper we study the superstring version of the exactly solvable string model constructed by Russo and Tseytlin. This model represents superstring theory in a curved spacetime and can be seen as a generalization of the Melvin background. We investigate D-branes in this model as probes of the background geometry by constructing the boundary states. We find that spacetime singularities in the model become smooth at high energy from the viewpoint of open string. We show that there always exist bulk (movable) D-branes by the effect of electric flux. The model also includes Nappi-Witten model as the Penrose limit and supersymmetry is enhanced in the limit. We examine this phenomenon in the open string spectrum. We also find the similar enhancement of supersymmetry can be occurred in several coset models.

Keywords: D-branes, Conformal Field Models in String Theory, Penrose limit and pp-wave background.

Contents

1. Introduction

Recently, many aspects of string theory have been uncovered. If we take the perturbative analysis of conformal field theory as an example, however, most of the discussions are restricted to the curved space not the curved spacetime. Thus we cannot completely answer how to do with the intriguing phenomena peculiar to curved spacetime such as black holes and spacetime singularities in the framework of string theory. It may seem natural to expect some stringy resolution of singularities as in orbifold theories. Motivated by this we would like to study the superstring version of the bosonic model [1] constructed[1] by Russo and Tseytlin.

[1] For recent discussions on conformal field theoretic approach to time dependent background see e.g. [2]-[6].

The best advantage of this model is that it is exactly solvable. Thus we can compute the closed string spectrum completely. Furthermore, this background has spacetime singularities in some parameter regions and thus will be suitable for the study of stringy analysis of spacetime singularities. As particular limits, this model includes the Melvin background [7]. The background is generally non-supersymmetric and includes closed string tachyons. One type of tachyons is the localized tachyon discussed recently in [8]–[13]. There is also another type of the tachyon which appears when the spacetime has singularities.

The main purpose of this paper is to investigate the geometry of the background by using D-brane probes. Especially, we construct explicit boundary states in this spacetime. As a result we will show that the geometry becomes non-singular at high energy from the viewpoint of open string even if there are apparent spacetime singularities. Our analysis is parallel with the previous studies on D-branes in the Melvin background ([14, 15, 16]). However, we will find several crucial differences from the results in the Melvin model. For example, we can always put D0-branes anywhere in this curved spacetime. Thus we can probe the geometry smoothly, while in the Melvin model we can construct D0-branes only at the origin for irrational values of parameters [15, 16].

This model also includes Nappi-Witten model [17] as its Penrose limit (pp-wave limit) [18] (for recent discussions on the duality between string on pp-waves and gauge theory see [19] and also [20]–[28]). Only in this background there are unbroken supersymmetries and thus the supersymmetry is enhanced by taking the Penrose limit of this model. Later we will analyze the similar enhancement does occur in several coset models. We will also discuss D-branes in Nappi-Witten model by employing our general results.

This paper is organized as follows. In section 2, we review the model [1] and show the detailed analysis of its supersymmetrization. In section 3, we construct the boundary states of D-branes in this model and compute the open string spectra. In section 4 , we consider the enhancement of supersymmetry in the Penrose limit of our model as well as the coset models [29, 30] from the viewpoint of both closed string and open string.

2. Exactly solvable superstring model of curved spacetime

Here we consider the supersymmetrization of the solvable model [1]. This solvable background describes the curved spacetime with four parameters R, q_+, α, β. In particular, it includes Nappi-Witten background [17] and the Melvin background [7] as specific limits. The explicit form of metric $G_{\mu\nu}$, NSNS B-field $B_{\mu\nu}$ and dilaton ϕ of this background is given by

$$\begin{aligned}
ds^2 &= -dt^2 + dy^2 + d\rho^2 + \\
&\quad + \frac{\rho^2}{1+\alpha\beta\rho^2}\left(d\varphi + (q_+ + \beta)dy - (q_- + \beta)dt\right)\left(d\varphi + (q_+ - \alpha)dy - (q_- + \alpha)dt\right), \\
B_{y\varphi} &= \frac{\alpha+\beta}{2}\frac{\rho^2}{1+\alpha\beta\rho^2}\,, \qquad B_{t\varphi} = \frac{\alpha-\beta}{2}\frac{\rho^2}{1+\alpha\beta\rho^2}\,, \\
B_{ty} &= \left(\frac{\alpha-\beta}{2}q_+ + \frac{\alpha+\beta}{2}q_- + \frac{\alpha^2+\beta^2}{2}\right)\frac{\rho^2}{1+\alpha\beta\rho^2}\,, \qquad e^{2(\phi-\phi_0)} = \frac{1}{1+\alpha\beta\rho^2}\,, \qquad (2.1)
\end{aligned}$$

where we introduced the parameters α, β, q_+ and q_-. We can change the value of q_- by shifting the field φ such that $\varphi \to \varphi + \lambda t$ and thus q_- is an auxiliary parameter. We also assume that the coordinate y is compactified and its radius is denoted by R. After the Kaluza-Klein compactification we can obtain a series of curved lorentzian backgrounds with electro-magnetic flux as discussed in [1].

If we assume the specific parameter region $\alpha\beta < 0$, then the background becomes singular[2] at $\rho_0 = 1/\sqrt{-\alpha\beta}$. However, we have the free field representation even in such a case as we will see and thus we include these singular cases in our analysis. Later we will discuss how these spacetime singularities affect closed string and open string.

2.1 World-sheet sigma model and its free field representation

The most important advantage of this model is that we can solve the model exactly by using the free field representation. This is explicitly shown in the bosonic theory [1]. Its extension to supersymmetric theory can be done in a rather straightforward way as we show below (this procedure is parallel with the Melvin model [7, 15] and the uniform magnetic field model [31]).

The background (2.1) is described by the following sigma-model

$$\begin{aligned} S_1 = \frac{1}{\pi\alpha'}\int d\sigma^2 \Big[&- \partial t\bar{\partial}t + \partial y\bar{\partial}y + \partial\rho\bar{\partial}\rho + \frac{\rho^2}{1+\alpha\beta\rho^2}(\partial\varphi + (q_+ + \beta)\partial y - (q_- + \beta)\partial t) \times \\ &\times \left(\bar{\partial}\varphi + (q_+ - \alpha)\bar{\partial}y - (q_- + \alpha)\bar{\partial}t\right) + \\ &+ \frac{\alpha'}{4}R^{(2)}\left(\phi_0 - \frac{1}{2}\ln(1+\alpha\beta\rho^2)\right)\Big], \end{aligned} \tag{2.2}$$

where $R^{(2)}$ is the Ricci scalar of the world sheet. Here we omitted the fermion terms because they can be simply obtained by using the superspace formalism as in [15]. The last term of (2.2) represents the dilaton coupling and below we will not write this explicitly.

We would like to show how to solve this model exactly by examining it in a covariant way without using the light-cone gauge. First we perform T-duality with respect to φ (see also [1, 15]) and obtain

$$\begin{aligned} S_2 = \frac{1}{\pi\alpha'}\int d\sigma^2 \Big[&(\partial u + \alpha\partial\tilde{\varphi})(\partial v + \beta\bar{\partial}\tilde{\varphi}) + \partial\rho\bar{\partial}\rho + \frac{1}{\rho^2}\partial\varphi\bar{\partial}\tilde{\varphi} + 2q_+(\partial y\bar{\partial}\tilde{\varphi} - \partial\tilde{\varphi}\bar{\partial}y) - \\ &- 2q_-(\partial t\bar{\partial}\varphi - \partial\varphi\bar{\partial}t)\Big], \end{aligned} \tag{2.3}$$

where we have defined $u = y - t$ and $v = y + t$. After we replace[3] u, v with U, V as follows

$$U = u + \alpha\tilde{\varphi}\ (= Y - T), \qquad V = v + \beta\tilde{\varphi}\ (= Y + T), \tag{2.4}$$

we can take the second T-duality on $\tilde{\varphi}$,

$$S_3 = \frac{1}{\pi\alpha'}\int d\sigma^2 \left[\partial U\partial V + \partial\rho\bar{\partial}\rho + \rho^2(\partial\varphi' + q_+\partial Y - q_-\partial T)(\bar{\partial}\varphi' + q_+\bar{\partial}Y - q_-\bar{\partial}T)\right]. \tag{2.5}$$

[2]Obviously, the string coupling e^{ϕ} shows the singular behavior. Furthermore, it is also easy to check that the curvature tensors of the metric and B-field diverge at the same points. If we take the T-duality, we have also the singularity at $1 + q_+(q_+ + \beta - \alpha)\rho^2 = 0$.

[3]Here we would like to notice that the time direction T of the free fields includes the winding number of $\tilde{\varphi}$. This produces an important stringy correction.

If we shift the angular coordinate such that $\varphi'' = \varphi' + q_+ Y - q_- T$, then we obtain the free field action

$$S_3 = \frac{1}{\pi\alpha'} \int d\sigma^2 [\partial U \bar{\partial} V + \partial X \bar{\partial} \bar{X}] , \tag{2.6}$$

where we defined $X = \rho e^{i\varphi''}$ and $\bar{X} = \rho e^{-i\varphi''}$. Then we can see that the fields U, V, X and $\bar{X}$ are all free fields. In this way we can reduce the original action (2.2) to the free field one (2.6) and thus we can solve the model exactly.

Next let us examine the mass spectrum. We define the angular momentum

$$\hat{J}_L = \frac{1}{2\pi i} \oint dz\ j_L(z) , \qquad \hat{J}_R = -\frac{1}{2\pi i} \oint d\bar{z}\ j_R(\bar{z}) , \tag{2.7}$$

where $j_{L,R}$ is defined as follows

$$\partial\tilde{\varphi} = -\rho^2 \partial \varphi'' + i\psi_L \bar{\psi}_L \equiv i\alpha' j_L , \qquad \bar{\partial}\tilde{\varphi} = \rho^2 \bar{\partial}\varphi'' - i\psi_R\bar{\psi}_R \equiv -i\alpha' j_R , \tag{2.8}$$

where we restored the contributions of fermions. Then we obtain the shifted periodicity of $\tilde{\varphi}$

$$\tilde{\varphi}(\tau, \sigma + 2\pi) = \tilde{\varphi}(\tau,\sigma) - 2\pi\alpha' \hat{J} . \tag{2.9}$$

By using the above and the canonical quantization of momenta in the original action (2.2), the momenta of free fields Y and T are given by

$$\begin{aligned} P^L_Y + P^R_Y &= 2\left(\frac{n}{R} - \left(q_+ + \frac{\beta-\alpha}{2}\right)\hat{J}\right) , \qquad P^L_Y - P^R_Y = 2\left(\frac{Rw}{\alpha'} - \frac{\alpha+\beta}{2}\right) , \\ P^L_T + P^R_T &= -2\left(E + \left(q_- + \frac{\alpha+\beta}{2}\right)\hat{J}\right) , \qquad P^L_T - P^R_T = (\alpha-\beta)\hat{J} , \end{aligned} \tag{2.10}$$

where $n/R, w$ and E represent the Kaluza-Klein momentum, winding number in the y direction and the energy.

The free fields $X, \bar{X}$ and $\psi_{L,R}, \bar{\psi}_{L,R}$ obey the following twisted boundary conditions

$$\begin{aligned} X(\tau,\sigma+2\pi) &= e^{2\pi i\gamma} X(\tau,\sigma) , \\ \psi_L(\tau,\sigma+2\pi) &= e^{2\pi i\gamma}\psi_L(\tau,\sigma) , \qquad \psi_R(\tau,\sigma+2\pi) = e^{2\pi i\gamma}\psi_R(\tau,\sigma) , \end{aligned} \tag{2.11}$$

where γ is defined by

$$\begin{aligned} \gamma = &\left(q_+ + \frac{\beta-\alpha}{2}\right) wR + \frac{\alpha+\beta}{2}\alpha' \left(\frac{n}{R} - \left(q_+ + \frac{\beta-\alpha}{2}\right)\hat{J}\right) + \\ &+ \frac{\beta-\alpha}{2}\alpha' \left(E + \left(q_- + \frac{\alpha+\beta}{2}\right)\hat{J}\right) . \end{aligned} \tag{2.12}$$

Now we can compute the spectrum of the string model. The spectrum is given by $H_c = 0$, where H_c is the closed string hamiltonian

$$\begin{aligned} H_c = &-\frac{\alpha'}{2}\left(E + \left(q_- + \frac{\alpha+\beta}{2}\right)\hat{J}\right)^2 + \frac{R^2}{2\alpha'}\left(w - \frac{\alpha'}{R}\frac{\alpha+\beta}{2}\hat{J}\right)^2 + \\ &+ \frac{\alpha'}{2R^2}\left(n - \left(q_+ + \frac{\beta-\alpha}{2}\right)R\hat{J}\right)^2 + \\ &+ \sum_{i=1}^{6} \frac{\alpha'}{2}(p^i)^2 - \frac{\alpha'}{8}(\alpha-\beta)^2\hat{J}^2 + \hat{N}_R + \hat{N}_L - \hat{\gamma}(\hat{J}_R - \hat{J}_L) , \end{aligned} \tag{2.13}$$

with the level matching condition

$$\hat{N}_R - \hat{N}_L - nw + [\gamma]\hat{J} = 0\,, \tag{2.14}$$

where we define $\gamma \equiv [\gamma] + \hat{\gamma}$; $[\gamma]$ denotes the integer part of γ. Even though this expression is the same as in [1], we took the fermions into account (see (2.7)) here. Note that if $\hat{\gamma} = 0$, we must add the contribution of zero modes of X to H_c.

2.2 Partition function

It is useful to compute the one-loop partition function of this string model. The amplitude can be most easily computed by the path-integral method in the light-cone gauge as was done in the Melvin model (see also [7, 10]). We show the result in Green-Schwarz formulation as follows (for the bosonic model see [1])

$$\begin{aligned} Z(R, q_+, \alpha, \beta) \;=\; & (2\pi)^{-7} V_7 R(\alpha')^{-5} \int \frac{(d\tau)^2}{(\tau_2)^6} \int (dC)^2 \sum_{w, w' \in \mathbb{Z}} \frac{\theta_1(\frac{\chi}{2}|\tau)^4 \theta_1(\frac{\tilde{\chi}}{2}|\bar{\tau})^4}{|\eta(\tau)|^{18} \theta_1(\chi|\tau)\theta_1(\tilde{\chi}|\bar{\tau})} \times \\ & \times \exp\left[-\frac{\pi}{\alpha' \tau_2}(4C\bar{C} - 2\bar{C}R(w' - w\tau) + 2CR(w' - w\bar{\tau})) \right], \end{aligned} \tag{2.15}$$

where V_7 denotes the infinite volume of $\mathbb{R}^7$ and we have defined

$$\chi = 2\beta C + q_+ R(w' - \tau w)\,, \qquad \tilde{\chi} = 2\alpha \bar{C} + q_+ R(w' - \bar{\tau} w)\,. \tag{2.16}$$

It is easy to see that the previous spectrum (2.13) in the operator formulation is reproduced by employing the Poisson resummation. If one uses the Jacobi identity

$$\theta_3(0|\tau)^3 \theta_3(\chi|\tau) - \theta_2(0|\tau)^3 \theta_2(\chi|\tau) - \theta_4(0|\tau)^3 \theta_4(\chi|\tau) = 2\theta_1 \left(\frac{\chi}{2} | \tau \right)^4, \tag{2.17}$$

then this explicitly represents the path-integral in the NS-R formulation with correct type-II GSO-projection.

Next we would like to mention the T-duality symmetry. The result is the same as in the bosonic model [1]. Thus we obtain the following results.

$$Z(R, \alpha, \beta, q_+) = Z\left(\frac{\alpha'}{R}, q_+, \beta - \alpha + q_+, \alpha \right) = Z\left(\frac{\alpha'}{R}, \alpha - \beta - q_+, -q_+, -\beta \right). \tag{2.18}$$

In other words, the model is invariant under the exchange of parameters

$$R \longleftrightarrow \frac{\alpha'}{R}\,, \qquad \frac{\alpha + \beta}{2} \longleftrightarrow \tilde{q} = \frac{2q_+ + \beta - \alpha}{2}\,, \qquad (\beta - \alpha = \text{fixed})\,. \tag{2.19}$$

We can also see that the superstring theory in the Melvin background [7] is included as a particular case $\beta = \alpha$, $q_- = -\beta$, $q_+ = q$. In particular this shows that this background includes the two-dimensional oribfolds $\mathbb{C}/\mathbb{Z}_N$ in type-II and type-0 string theory [10].

For generic values of parameters as we can see from (2.15), the partition function does not vanish. This means that this model is non-supersymmetric in general.

2.3 Penrose limit and Nappi-Witten model

We can also see that the sigma model (2.2) is equivalent to the (compactified) Nappi-Witten model [17] (see also [32, 27] and references their in)

$$S = \frac{1}{\pi\alpha'}\int d\sigma^2[\partial u\bar{\partial}v + \beta\rho^2\partial u\bar{\partial}\phi + \rho^2\partial\phi\bar{\partial}\phi]\,, \tag{2.20}$$

if we set $\alpha = q_\pm = 0$ (or $\beta = q_\pm = 0$) as shown in [1]. The spectrum of this model is given by (see also [33])

$$\frac{\alpha'}{2}E^2 + \alpha'\beta E\hat{J}_R = \frac{R^2}{2\alpha'}w^2 + \frac{\alpha'}{2R^2}n^2 + \beta wR\hat{J}_R - \alpha'\beta\frac{n}{R}\hat{J}_R + \hat{N}_L + \hat{N}_R\,, \tag{2.21}$$

where we assumed $0 \leq \gamma < 1$. Note that this spectrum is invariant under the simple T-duality transformation $R \to \alpha'/R$. We can also compute its partition function as a limit of (2.15)

$$Z(R,\beta) = (\alpha')^{-7}V_7R\tilde{V}\int\frac{d\tau^2}{\tau_2^{11/2}}e^{-\frac{\pi R^2}{\alpha'\tau_2}|w'-w\tau|^2}\frac{\theta_1(\beta R(w'-\tau w)/2|\tau)^4\;\theta_1(0|\bar{\tau})^4}{\eta(\tau)^9\eta(\bar{\tau})^{12}\;\theta_1(\beta R(w'-\tau w)|\tau)}\,, \tag{2.22}$$

where $\tilde{V}$ denotes a divergent factor coming from the extra zero-modes along $X, \bar{X}$ direction. Note that this partition function does vanish[4] because $\theta_1(0|\tau) = 0$ and this is consistent with the fact that Nappi-Witten background preserves partial supersymmetries [32, 33]. We can also see that this is the only case of vanishing partition function. Note that this special background satisfies the condition $B_{y\varphi} = B_{t\varphi}$. It would also be useful to see that the partition function (2.22) shows that the background can be regarded as an orbifold with respect to only left-moving sectors if we assume the fractional case $\beta R = k/N \in \mathbb{Q}$.

These facts can be understood by taking the Penrose limit [18, 34] of (2.1)

$$\tilde{u} = u\,, \qquad \Omega^2\tilde{v} = v\,, \qquad \Omega\tilde{\rho} = \rho\,, \qquad \text{and} \qquad \Omega \longrightarrow 0\,, \tag{2.23}$$

which leads to the following (rescaled) metric

$$(ds')^2 = \Omega^{-2}(ds)^2 = d\tilde{u}d\tilde{v} + d\tilde{\rho}^2 + \tilde{\rho}^2(d\varphi + (q_1+\beta)d\tilde{u})(d\varphi + q_1 d\tilde{u})\,, \tag{2.24}$$

where we defined $q_\pm = q_1 \pm q_2$. Thus we come back to the specific model $\alpha = q_2 = 0$. Since the Penrose limit decompactifies the circle in the y direction, we can set $q_1 = 0$ by the redefinition of coordinates and we finally obtain the Nappi-Witten model. Notice that after we take the Penrose limit of the non-supersymmetric model, we eventually find the supersymmetric background. Later we will also give some other examples of similar supersymmetry enhancement in the Penrose limit.

[4] If we take the non-compact limit $R = \infty$ (or equally $R = 0$ by T-duality), then the partition function is equivalent to that in flat space [32]. However this does not imply the equivalence between both models since the interactions are obviously different. Later we will find the open string spectrum in the Nappi-Witten model which differs from that in flat space.

2.4 Closed string tachyons and spacetime singularity

In general the background (2.1) is non-supersymmetric as can be seen from the fact that the partition function of this background does not vanish. Thus it is very interesting to discuss the instability of the model. In the bosonic string model some relevant discussions were given in [1]. Here we would like to consider the tachyonic instability in the superstring model. We identify a tachyonic mode with an excitation which has a non-zero imaginary part[5] of energy E in the spectrum (2.13).

In this model tachyons arises due to two reasons. The first reason is the presence of the term which is proportional to $\hat{\gamma}$. The effect of this is the existence of localized tachyons[6] [8, 9, 11, 12, 13] and they have already appeared in the Melvin background [7] $\alpha = \beta$ (for discussions of closed string tachyon condensation in Melvin background see e.g. [9]–[13] and [36, 37, 38]). This type of tachyonic modes has non-zero values of n or w.

The second reason is that there is the term which is proportional to $-\hat{J}^2$ in the spectrum (2.13). This leads to tachyons even if we set $n = w = 0$. Let us assume $\tilde{q} \equiv q_+ + \frac{\beta-\alpha}{2} = 0$ for simplicity and consider gravitons ($\hat{N}_L = \hat{N}_R = 0$) which has the spin $\hat{J}_L = \hat{J}_R = \pm 1/2$. Then we can see that the mode is tachyonic if and only if $\alpha\beta < 0$. As in the bosonic case [1] we can speculate that this instability occurs due to the spacetime singularity of the background (2.1) at the fixed ρ_0 such that $1 + \alpha\beta\rho_0^2 = 0$. Notice that this tachyon field is not localized one since the corresponding off-shell (tachyonic) vertex operator has the vanishing value of $E + (q_- + \frac{\alpha+\beta}{2})\hat{J}$. We can also show that such tachyons do not exist in the RR-fields.

One of the most interesting questions is what background this unstable model will decay into. If we remember the arguments that the Melvin background will become the ordinary type-II string theory [13, 37] after the closed string tachyon condensation, then it seems to be natural to identify the decay product of the general non-supersymmetric background $\alpha\beta > 0$ with the Nappi-Witten background. In the case of $\alpha\beta < 0$ (singular spacetime) we cannot even speculate the answer because we must consider the condensation of the "new type" (not localized) tachyons. We would like to leave these as future problems. Instead we will later discuss whether we can probe this spacetime singularity by using D-branes.

2.5 Higher-dimensional generalizations

It is also possible to construct higher-dimensional models. For example, we can generalize the action in the Green-Schwarz formalism as in the case of Melvin background [10, 11] such that there are $n \geq 2$ angular coordinates φ_i $(i = 1, \ldots, n)$ by introducing $3n$ parameters α_i, β_i and q_{+i}. Then the partition function ($n = 4$ case) is given by replacing the theta-function part of (2.15) with

$$\sum_{w,w' \in \mathbb{Z}} \frac{\left(\sum_{a=2}^{4}(-1)^{a+1}\prod_{i=1}^{4}\theta_a(\chi_i|\tau)\right)\left(\sum_{b=2}^{4}(-1)^{b+1}\prod_{j=1}^{4}\theta_b(\tilde{\chi}_j|\tau)\right)}{\prod_{i=1}^{4}\theta_1(\chi_i|\tau)\prod_{i=1}^{4}\theta_1(\tilde{\chi}_i|\tau)}\,. \tag{2.25}$$

[5] Since in this case $\mathrm{Im}\,E \neq 0$ the value of γ also becomes imaginary, the twisted boundary condition (2.11) may be related to the lorentzian orbifold discussed in [2]–[5] and [35].

[6] For rational values of parameters [10] the existence of bulk tachyons (as in type-0 theory) is also possible.

We can see that there exist supersymmetric models which have the vanishing vacuum amplitude (e.g. $\sum_i(\pm\chi_i) = 0$) in the same way as in [10, 11].

3. Boundary states in curved spacetime

Here we consider D-branes in the solvable model (2.2) by using the boundary state formalism. This will give the generalization of the arguments on D-branes in two parameter Melvin background [15] ($\alpha = \beta$). See also [14] for an independent analysis of D-branes in the one parameter (Kaluza-Klein Melvin) model.

Here we consider the general case $\alpha \neq \beta$ and a D-brane in this model can be regarded as a probe in the curved spacetime. Finally we will find the following two novel facts.

To begin with, there is the term $-\frac{(\alpha-\beta)^2}{8}\alpha'\hat{J}^2$ in the closed string hamiltonian (2.13) and this changes the open string hamiltonian. In the case of D0-branes in the free field representation, for instance, we will see that this effect leads to the term proportional to $\sin^2[\frac{\pi(\alpha-\beta)}{2}\alpha' E]$ in the open string hamiltonian. Though we can interpret this in terms of the curved metric in the low energy limit, this includes a stringy correction in the high energy region. We will see that this stringy correction makes the open string spectrum smooth and thus we cannot see the spacetime singularity in the high energy region.

The second result is that we can always move D0-branes by performing the Lorentz boost in the direction Y away from the origin when $\alpha \neq \beta$. This contrasts strikingly with the Melvin model ($\alpha = \beta$). The latter does not allow movable D0-branes (bulk D0-branes) if the value of $\beta\alpha'/R$ is irrational [15]. This means that there exist the bulk D-branes for any parameters if $\alpha \neq \beta$. In the original coordinate picture, this boost corresponds to adding the electric flux. And then this flux stabilizes the bulk D-branes.

3.1 Boundary conditions

Let us find the consistent boundary conditions. If we consider them in the free field representation $(T, Y, X, \bar{X})$, Neumann and Dirichlet boundary condition are obviously allowed (later we will discuss D-branes in the original coordinates to investigate their geometrical properties.)

The boundary conditions in the T and Y direction are given by

$$T : \begin{cases} \text{Neumann: } \partial_\tau T|_{\tau=0}|B\rangle = 0 \longrightarrow \left[2E + (2q_- + \alpha + \beta)\hat{J}\right]|B\rangle = 0\,, \\ \text{Dirichlet: } \partial_\sigma T|_{\tau=0}|B\rangle = 0 \longrightarrow (\beta - \alpha)\hat{J}|B\rangle = 0, \end{cases} \tag{3.1}$$

$$Y : \begin{cases} \text{Neumann: } \partial_\tau Y|_{\tau=0}|B\rangle = 0 \longrightarrow \left[\frac{2n}{R} - (2q_+ + \beta - \alpha)\hat{J}\right]|B\rangle = 0\,, \\ \text{Dirichlet: } \partial_\sigma Y|_{\tau=0}|B\rangle = 0 \longrightarrow \left[-\frac{2Rw}{\alpha'} + (\alpha + \beta)\hat{J}\right]|B\rangle = 0\,, \end{cases} \tag{3.2}$$

where the coordinates (σ, τ) represent the world-sheet of the closed string channel. In the most part of this paper, we consider D-branes which satisfy the Dirichlet-Dirichlet boundary condition in the $(X, \bar{X})$ directions since they can be used as probes of the background. The D-branes of Neumann-Neumann boundary conditions can also be examined similarly.[7] In

[7] Though Neumann-Dirichlet boundary condition can also be allowed, we will not discuss this case in the present paper.

these cases, we can verify

$$\hat{J}|B\rangle = \hat{J}_0|B\rangle\,, \tag{3.3}$$

where $|B\rangle$ is the boundary state and $\hat{J}_0$ is the bosonic zero mode contribution to the angular momentum $\hat{J}$.

Since we are interested in D-branes which are not localized in the time like direction, below we mainly assume that the field T satisfies Neumann boundary condition. In the last of this section we will briefly mention D-brane instantons. Thus we would like to show the detailed analysis of D0-branes and D1-branes whose boundary conditions are $(T,Y,X) = (N,D,D)$ and $(T,Y,X) = (N,N,D)$, where "N" means Neumann and "D" Dirichlet. The D2-branes $(T,Y,X) = (N,D,N)$ and D3-branes $(T,Y,X) = (N,N,N)$ can be treated almost in the same way as D0 and D1-branes.

3.2 Bulk D-branes

Now we consider the boundary conformal field theory of bulk D0 and D1-branes in the free field representation of boundary states. The term "bulk" means that the brane can leave from the origin $\rho = 0$ and thus it can probe the geometry of the spacetime. There are also D-branes which are fixed at the origin. We call the latter "fractional" as in the orbifold theory [39, 40]. Generally, a bulk D-brane is consist of finite numbers of fractional D-branes. We show the detailed analysis only for bulk D0-branes since bulk D1-branes can be obtained by taking the T-duality (2.19) in the Y direction. The fractional D-branes are also discussed in the appendix A.

Bulk D0-brane. The boundary conditions $(T,Y) = (N,D)$ require that the zero modes should satisfy

$$E + \left(q_- + \frac{\alpha+\beta}{2}\right)\hat{J} = 0\,, \qquad Rw - \frac{(\alpha+\beta)}{2}\alpha'\hat{J} = 0\,. \tag{3.4}$$

By using these conditions, we can write the closed string hamiltonian (see (2.13)) which acts on $|B\rangle$ as

$$\begin{aligned} 2H_c = {} & \frac{\alpha'}{2}(p^i)^2 + \frac{\alpha'}{2R^2}\left[n - \left(q_+ + \frac{\beta-\alpha}{2}\right)R\hat{J}_0\right]^2 - \\ & - \frac{(\alpha-\beta)^2}{8}\alpha'\hat{J}_0^2 + \hat{N}_L + \hat{N}_R + \hat{\gamma}(\hat{J}_R - \hat{J}_L)\,, \end{aligned} \tag{3.5}$$

where γ is

$$\gamma = \frac{\alpha'}{2}(\alpha+\beta)\frac{n}{R}\,. \tag{3.6}$$

Notice that γ depends only on n, not on $\hat{J}_0$.

When $\alpha'(\alpha+\beta)/2R$ is rational ($\equiv k/N \in \mathbb{Q}$),[8] there are bulk D0-branes as in the case of the Melvin background.[9] We can find that a bulk D0-brane at $\rho = 0$ consists of N fractional D0-branes whose positions in the Y direction are given by $y_0, y_0 + 2\pi R/N, \ldots$ (see appendix A).

[8] We assume that k and N are coprime.

[9] As we can see from eq. (3.6), we have only to change β to $\alpha + \beta/2$.

Then let us move the bulk D0-brane from $\rho = 0$. In the case of $\rho \neq 0$, the states of $\hat{J}_0 \neq 0$ are allowed. First, from eq. (3.4) we obtain the constraint

$$\hat{J}_0 \in N\mathbb{Z}, \tag{3.7}$$

and thus we find that the bulk D0-brane at $\rho \neq 0$ should consist of N fractional D0-branes located at N points $X = \rho,\ \rho e^{2\pi i/N}, \ldots,$ in the X direction such that the bulk D0-brane is invariant under the action $X \to e^{2\pi i/N}X$. We can also understand[10] this fact from the explicit form of the boundary state (appendix B).

Now let us see whether the bulk D0-brane can exist at $\rho \neq 0$ in this case. In order to check this we must examine the Cardy's condition [42]. The vacuum amplitude (assuming k=even for simplicity)

$$\mathcal{A} = \int ds \langle B| \frac{\alpha'}{2} e^{-2sH_c} |B\rangle\,, \tag{3.8}$$

is given by

$$\mathcal{A}_{\text{bulk}} = \frac{\alpha' V_0}{8\pi R}\left(\frac{NT_0}{2}\right)^2 \int ds\, (2\pi\alpha' s)^{-3}\eta(\tau)^{-12}[\theta_3(0|\tau)^4 - \theta_4(0|\tau)^4 - \theta_2(0|\tau)^4]\, Z_0(\tau)\,, \tag{3.9}$$

where V_0 denotes the volume in the time direction and we defined $T_0 = \sqrt{\pi}(2\pi\sqrt{\alpha'})^3$. We have also written the zero-mode part of the amplitude as $Z_0(\tau)$, $\tau \equiv is/\pi$ and it is given by

$$Z_0 = \langle B_0| e^{-s\left[\frac{\alpha' \vec{k}^2}{2} - \frac{(\alpha-\beta)^2}{8}\alpha' \hat{J}_0^2 + \frac{\alpha'}{2R^2}(n - \tilde{q}R\hat{J}_0)^2\right]} |B_0\rangle\,, \tag{3.10}$$

where we defined $\tilde{q} = q_+ + \frac{\beta-\alpha}{2}$ (see (2.19)) and $\vec{k}$ is the momentum in the X direction. Notice that we can find that the above vacuum amplitude (3.9) does vanish and Bose-Fermi degeneracy occurs. Below we first assume $\alpha + \beta = 0$ $(k = 0, N = 1)$ and later consider general cases. Then we can write the zero-mode part of the boundary state as

$$|B_0\rangle = \sum_n \frac{1}{(2\pi)^2}\int (d\vec{k})^2\ e^{i\vec{k}\cdot\vec{x}} |\vec{k}\rangle \otimes |n\rangle\,, \tag{3.11}$$

where $\vec{x} = (\rho\cos\phi, \rho\sin\phi)$ is the position of the D0-brane in the X direction and the y-momentum eigenstate $|n\rangle$ is normalized as $\langle n|n'\rangle = \delta_{nn'}$. Next we perform the modular transformation $(s = \pi/t)$ of this amplitude. After the the Poisson resummation of n and the Gauss integration of the open string energy E, we obtain

$$\begin{aligned}
Z_0 &= \sum_w \sqrt{\frac{2R^2 t}{\alpha'}}\sqrt{\frac{2\pi\alpha'}{s}}\int (dE)\ \langle B_0'|\ e^{\frac{2\pi^2\alpha'}{s}E^2 - \frac{s\alpha'\vec{k}^2}{2} - 2\pi t\frac{R^2 w^2}{\alpha'} + i\pi((\alpha-\beta)\alpha' E + 2\tilde{q}Rw)\hat{J}_0} |B_0'\rangle \\
&= \sum_w \frac{2Rt}{(2\pi)^2}\int (d\vec{k})^2 (dE) e^{\frac{2\pi^2\alpha'}{s}E^2 - \frac{s\alpha'\vec{k}^2}{2} - 2\pi t\frac{R^2 w^2}{\alpha'}} e^{i\vec{k}\Delta\vec{x}}\,,
\end{aligned} \tag{3.12}$$

[10] The world-volume theory of such bulk D-branes seems to be described by a sort of quiver gauge theory with monodromy discussed in [12, 41]. We thank Y. Sugawara for pointing out this to us.

where we defined $|B'_0\rangle$ as the zero mode part in the X direction. Note that we have employed the fact that the operator $e^{i\theta \hat{J}_0}$ acts as the rotation $k_1 + ik_2 \to e^{i\theta}(k_1 + ik_2)$ (or equally $x_1 + ix_2 \to e^{i\theta}(x_1 + ix_2)$). The length of $\vec{\Delta x}$ is given by

$$|\vec{\Delta x}| = \left| 2\rho \sin\left(\frac{\pi(\alpha-\beta)}{2}\alpha' E + \pi\tilde{q}Rw\right)\right| . \tag{3.13}$$

Then after we integrate out $\vec{k}$, the amplitude becomes

$$Z_0 = \sum_w \frac{2Rt^2}{\pi\alpha'}\frac{1}{2\pi}\int dE e^{-2\pi t H_o} , \tag{3.14}$$

where H_o is given by

$$H_o = -\alpha' E^2 + \frac{R^2 w^2}{\alpha'} + \frac{\rho^2}{\pi^2\alpha'}\sin^2\left(\frac{\pi(\alpha-\beta)}{2}\alpha' E + \pi\tilde{q}Rw\right) . \tag{3.15}$$

In order to generalize this result for $(\alpha+\beta)\alpha'/2R = k/N$, we have only to change eq. (3.14) and (3.15) about a few points (see appendix B for detail). Finally, we obtain the open string 1-loop amplitude

$$A_{\mathrm{bulk}} = 2\times\int_0^\infty \frac{dt}{2t}\mathrm{Tr}_{NS-R}\left[\frac{1+(-1)^F}{2}e^{-2\pi t H_o}\right] , \tag{3.16}$$

where the open string hamiltonian H_o is written as

$$H_o = -\alpha' E^2 + \frac{R^2 w^2}{\alpha' N^2} + \frac{\rho^2}{\pi^2\alpha'}\sin^2\left[\frac{(\alpha-\beta)}{2}\pi\alpha' E + \pi\tilde{q}R\frac{w}{N} + \frac{\pi m}{N}\right] + \hat{N} . \tag{3.17}$$

The trace $\mathrm{Tr}_{NS-R} \equiv \mathrm{Tr}_{NS} - \mathrm{Tr}_R$ means

$$\mathrm{Tr}_{NS-R} = \frac{V_0}{2\pi}\int dE \sum_{m=0}^{N-1}\sum_{w=-\infty}^{\infty} \cdots . \tag{3.18}$$

We can easily see from eq. (3.16) and (3.18) that the result satisfies the Cardy's condition.[11]

Bulk D1-brane. By using T-duality (2.19) in the Y direction, we can obtain the result of bulk D1-brane. The open string hamiltonian is given by

$$H_o = -\alpha' E^2 + \alpha'\frac{n^2}{R^2 N^2} + \frac{\rho^2}{\pi^2\alpha'}\sin^2\left[\frac{(\alpha-\beta)}{2}\pi\alpha' E + \pi\frac{(\alpha+\beta)}{2R}\alpha'\frac{n}{N} + \frac{\pi m}{N}\right] + \hat{N} , \tag{3.19}$$

where we defined $(q_+ + \frac{\beta-\alpha}{2})R = k/N$.

[11] If we consider the limit $\rho \to 0$,

$$H_o \longrightarrow -\alpha' E^2 + \frac{R^2 w^2}{\alpha' N^2} + \hat{N} , \qquad \mathrm{Tr}_{NS-R} \longrightarrow \frac{NV_0}{2\pi}\int dE \sum_{w=-\infty}^{\infty} \cdots .$$

The above calculations are consistent with the results at $\rho = 0$ (A.10) and (A.11) analyzed in the different way.

3.3 D-branes as probes

Here we will further study a bulk Dp-brane (p=0,1) in this model so that we can probe the geometry for the fixed values of ρ.

Let us first consider the bulk D1-brane and assume $\tilde{q}=0$ for simplicity. If we consider the low energy limit (or equally $\alpha' \to 0$ limit)

$$E \ll \frac{1}{(\alpha-\beta)\alpha'}\,, \qquad p_y = \frac{n}{R} \ll \frac{1}{(\alpha+\beta)\alpha'} \tag{3.20}$$

then we can expand eq. (3.19) as

$$H_o = -\alpha'\left[1-\frac{(\alpha-\beta)^2}{4}\rho^2\right]E^2+\alpha'\frac{(\alpha^2-\beta^2)}{2}\rho^2 E p_y+\alpha'\left[1+\frac{(\alpha+\beta)^2}{4}\rho^2\right]p_y^2+\cdots\,. \tag{3.21}$$

This result can be interpreted as the expression $H_o = \alpha' G^{\mu\nu}p_\mu p_\nu + \cdots$ and is indeed consistent with the original metric (see (2.1))

$$G^{tt} = -1+\frac{(\alpha-\beta)^2}{4}\rho^2\,, \qquad G^{ty} = \frac{(\alpha^2-\beta^2)}{4}\rho^2\,, \qquad G^{yy} = 1+\frac{(\alpha+\beta)^2}{4}\rho^2\,. \tag{3.22}$$

On the other hand, in the high energy region we have a non-trivial α' correction as can be seen from the sin factor in (3.19), which comes from the winding number of time direction. This leads to the intriguing fact that there always exists a non-tachyonic pole[12] ($\mathrm{Im}\, E = 0$) as the solution of $H_o = 0$ for any open string modes in spite of the presence of spacetime singularities and closed string tachyons. Note that if the low energy expression (3.21) held for any E, the spectrum would include only open string tachyons if $1+\alpha\beta\rho^2 < 0$, which just corresponds to the existence of spacetime singularity as discussed in section 2.4. We can also see that if we assume the large value of ρ, there can be many (non-tachyonic) poles in the low energy region, while the behavior at high energy approaches that in the flat space.

Any way we have observed that the stringy effect seems to make the spacetime singularity smooth at high energy from the viewpoint of open string.

A similar result can be obtained for the open strings on D0-branes and the low energy limit of the spectrum is given as follows

$$H_o = -\alpha' E^2\left(1-\frac{(\alpha-\beta)^2}{4}\rho^2\right)+\cdots\,. \tag{3.23}$$

Again the spacetime singularity cannot be seen in the high energy region.

3.4 Boosted D-branes

It is also interesting to consider a bulk D0-brane which is boosted in the Y direction since we are discussing the curved spacetime (for a review of boundary states including boosted

[12]Here we should note that tachyonic poles ($\mathrm{Im}\, E \neq 0$) can also exist in the equation $H_o = 0$. However, we cannot answer whether these poles are physically relevant.

branes see [43]). The boundary condition is given by

$$
\begin{aligned}
\partial_\tau(T+vY)|_{\tau=0}|B\rangle = 0: &\qquad 2E+(2q_-+\alpha+\beta)\hat{J}_0 = v\left[\frac{2n}{R}-(2q_++\beta-\alpha)\hat{J}_0\right],\\
\partial_\sigma(vT+Y)|_{\tau=0}|B\rangle = 0: &\qquad v(\alpha-\beta)\hat{J}_0 = -\frac{2Rw}{\alpha'}+(\alpha+\beta)\hat{J}_0\,, \qquad (3.24)
\end{aligned}
$$

where v is the velocity of D0-brane. Then the closed string hamiltonian changes into

$$
\begin{aligned}
2H_c =& \frac{\alpha'}{2}(p^i)^2+(1-v^2)\frac{\alpha'}{2R^2}\left[n-\left(q_++\frac{\beta-\alpha}{2}\right)R\hat{J}_0\right]^2-(1-v^2)\frac{(\alpha-\beta)^2}{8}\alpha'\hat{J}_0^2+\\
&+\hat{N}_L+\hat{N}_R-\hat{\gamma}(\hat{J}_R-\hat{J}_L)\,, \qquad (3.25)
\end{aligned}
$$

where γ is given by

$$
\gamma = \frac{\alpha'}{2R}\left[\alpha+\beta+v(\beta-\alpha)\right]n\equiv\gamma_{v0}n\,. \qquad (3.26)
$$

Thus we find that in the case of $\alpha\neq\beta$, we can always make the value of γ_{v0} rational by tuning the velocity, even if $(\alpha+\beta)\alpha'/2R$ is irrational. This means that bulk D0-branes always exist for any value of $(\alpha+\beta)\alpha'/2R$. This is crucially different from the result in Melvin model ($\beta=\alpha$), where a bulk D0-brane does not exist when $\beta\alpha'/R$ is irrational.

Now let us check that boosted bulk D0-branes satisfy the Cardy's condition.[13] First, when $\gamma_{v0}=k/N$ and k =even, the vacuum amplitude (3.9) changes to

$$
\mathcal{A}_{\text{bulk}} = \sqrt{1-v^2}\times\frac{\alpha' V_0}{8\pi R}\left(\frac{NT_0}{2}\right)^2\int ds\, Z_0(v,s)\cdots\,, \qquad (3.27)
$$

where the factor $\sqrt{1-v^2}$ is due to the Lorentz contraction. But this factor is absorbed[14] when we use the Poisson resummation formula

$$
\begin{aligned}
&\sqrt{1-v^2}\sum_n \exp\left[-\frac{\alpha'(1-v^2)}{2R^2t}(n-\tilde{q}R\hat{J}_0)^2\right]=\\
&=\sqrt{\frac{2R^2t}{\alpha'}}\sum_w \exp\left[-\frac{2\pi R^2 t}{\alpha'(1-v^2)}w^2+2\pi i\tilde{q}Rw\hat{J}_0\right]. \qquad (3.28)
\end{aligned}
$$

Then, we can find from eq. (3.28) that the boosted bulk D0-branes also satisfy the Cardy's condition. Finally, the open string hamiltonian changes to

$$
\begin{aligned}
H_o =& -\alpha' E^2+\frac{R^2w^2}{\alpha'(1-v^2)N^2}+\frac{\rho^2}{\pi^2\alpha'}\sin^2\left[\frac{\sqrt{1-v^2}(\alpha-\beta)}{2}\pi\alpha' E+\pi\tilde{q}R\frac{w}{N}+\frac{\pi m}{N}\right]+\\
&+\hat{N}\,. \qquad (3.29)
\end{aligned}
$$

The multiplicity of the open string channel is the same as in the previous case (3.18). Note that the Lorentz contraction factor $\sqrt{1-v^2}$ does not appear in the open string 1-loop trace (3.18) because the open string energy E is of the original coordinates picture.

[13]We show that boosted fractional D0-branes satisfy the Cardy's condition in appendix A.

[14]We set $N=1$ in eq. (3.28) for simplicity. For general values of N, v-dependence is the same as in $N=1$.

If we take the T-duality (2.19), a boosted D0-brane is transformed into a D1-brane with electric flux f. The boundary condition is written as follows

$$[\partial_\tau T + if\partial_\sigma Y]|_{\tau=0}|B\rangle = 0\,, \qquad [\partial_\tau Y + if\partial_\sigma T]|_{\tau=0}|B\rangle = 0\,. \tag{3.30}$$

The boundary state analysis shows that the condition of moving away from the origin is given by

$$\left[q_+ + (1+f)\frac{(\beta-\alpha)}{2}\right] R \in \mathbb{Q}\,. \tag{3.31}$$

Again we can choose the flux f such that the D1-brane can move around.

Finally, we obtain the open string hamiltonian of D1-brane

$$\begin{aligned} H_o = & -\alpha' E^2 + \frac{\alpha' n^2}{(1-f^2)R^2N^2} + \frac{\rho^2}{\pi^2\alpha'}\sin^2\left[\frac{\sqrt{1-f^2}(\alpha-\beta)}{2}\pi\alpha' E + \right. \\ & \left. + \pi\frac{(\alpha+\beta)}{2R}\alpha'\frac{n}{N} + \frac{\pi m}{N}\right] + \hat{N}\,, \end{aligned} \tag{3.32}$$

where we defined $R[q_+ + \frac{(1+f)}{2}(\beta-\alpha)] = k/N$.

3.5 Flux stabilizations of D-branes

Up to now we have considered D-branes in the free field theory. Even though this viewpoint is the most convenient for calculations, it is better for the study of geometric structure of these D-branes to examine in terms of the original coordinate (ρ,φ,t,y). This analysis can be done by extending the method in [16], where D2-branes in Melvin background were shown to be stabilized by the magnetic flux $B_{y\varphi} + F_{y\varphi}$. In our curved spacetime model we will see the electric flux also plays an important role. This explains the novel fact that we can move the D2-branes away from the origin for any value of α,β by adding the electric flux, which we have already found in the boundary state analysis.

The boundary condition in terms of the original coordinate can be obtained by the transformation of fields under the T-duality (see also [15])

$$\begin{aligned} \partial\tilde{\varphi} &= \frac{\rho^2}{1+\alpha\beta\rho^2}\left[-\partial\varphi - (q_+ + \beta)\partial y + (q_- + \beta)\partial t\right], \\ \bar{\partial}\tilde{\varphi} &= \frac{\rho^2}{1+\alpha\beta\rho^2}\left[\bar{\partial}\varphi + (q_+ - \alpha)\bar{\partial} y - (q_- + \alpha)\bar{\partial} t\right], \\ \partial\tilde{\varphi} &= -\rho^2\partial\varphi''\,, \qquad \bar{\partial}\tilde{\varphi} = \rho^2\bar{\partial}\varphi''\,. \end{aligned} \tag{3.33}$$

The open string boundary condition of a D0-brane (including boosted ones) is given by

$$\partial_1(T+vY) = 0\,, \qquad \partial_2(vT+Y) = 0\,, \qquad \partial_2\varphi'' = 0\,, \tag{3.34}$$

where the coordinates (σ_1,σ_2) represent the world sheet of the open string channel. Then by applying (3.33) and comparing the mixed boundary condition

$$G_{\mu\nu}\partial_1 X^\nu + i(B_{\mu\nu} + F_{\mu\nu})\partial_2 X^\nu = 0\,, \tag{3.35}$$

we can identify the D-brane as a D2-D0 bound state wrapped on the two-dimensional torus in the directions of y and φ. In other words, this corresponds to D2-branes with the following the electric and magnetic flux

$$F_{\hat{\varphi}t} = \frac{2v}{\alpha+\beta+v(\beta-\alpha)} \equiv vh\,, \qquad F_{\hat{\varphi}y} = \frac{2}{\alpha+\beta+v(\beta-\alpha)} = h\,, \tag{3.36}$$

where we define the coordinate $\hat{\varphi} = \varphi + (q_+ + \frac{\beta-\alpha}{2})y - (q_- + \frac{\alpha+\beta}{2})t$.

The important point is that the flux quantization $\frac{1}{4\pi^2\alpha'}\int_{\mathbb{T}^2} \mathrm{Tr}\, F \in \mathbb{Z}$ can be easily satisfied by choosing the value of velocity v suitably if $\alpha \neq \beta$. This makes a striking contrast with the D-branes in Melvin background ($\beta = \alpha$). This fact is consistent with the boundary state analysis since the twisting parameter given by $\gamma = \alpha' \frac{\alpha+\beta+v(\beta-\alpha)}{2}\frac{n}{R}$ should always be rational numbers. The quantization condition is given by

$$h = \frac{\alpha' N}{kR}\,, \tag{3.37}$$

where k and N represent the number of D2-branes and D0-branes, respectively.

We can also check that the energy of D2-brane wrapped on the torus by using DBI-action is independent of ρ and reproduces the mass of D0-branes as we show below.

The matrix $G+B+F$ in the $(t, y, \hat{\varphi})$ coordinate system is given by

$$(G+B+F)_{\mu\nu} = \frac{1}{H}\begin{pmatrix} -1-\frac{(\alpha+\beta)^2}{4}\rho^2 & \frac{(\beta^2-\alpha^2)}{4}\rho^2 & \frac{(\alpha-\beta)}{2}\rho^2 - vhH \\ \frac{(\beta^2-\alpha^2)}{4}\rho^2 & 1-\frac{(\alpha-\beta)^2}{4}\rho^2 & \frac{\alpha+\beta}{2}\rho^2 - hH \\ -\frac{(\alpha-\beta)}{2}\rho^2 + vhH & -\frac{\alpha+\beta}{2}\rho^2 + hH & \rho^2 \end{pmatrix}, \tag{3.38}$$

where we defined $H \equiv 1 + \alpha\beta\rho^2$. By applying (3.37), we can calculate the DBI action of the D2-D0 bound state

$$\begin{aligned} S &= \int dt \int dy \int d\hat{\varphi} \frac{e^{-\phi}}{4\pi^2(\alpha')^{3/2}}\, \mathrm{Tr}\sqrt{-\det(G+B+F)} \\ &= \frac{V_0 k R e^{-\phi_0}}{(\alpha')^{3/2}}\sqrt{-H\det(G+B+F)}\,, \end{aligned} \tag{3.39}$$

where we used the relations, $\phi = \phi_0 - \frac{1}{2}\ln H$. Now we can calculate from eq. (3.38)

$$-H\det(G+B+F) = \rho^2\left[\left(\frac{\alpha+\beta+v(\beta-\alpha)}{2}\right)h - 1\right]^2 + (1-v^2)h^2\,. \tag{3.40}$$

Then we find that the dependence of (3.40) on ρ vanishes only when the specific value (3.36) of the flux is satisfied. Finally we obtain by using the relation (3.37)

$$S = \sqrt{1-v^2}V_0 \times NT_{D0}\,, \qquad \left(T_{D0} \equiv e^{-\phi_0}\alpha'^{-1/2}\right). \tag{3.41}$$

This is the action of the bulk D0-brane in the free field picture (including the Lorentz contraction factor).

Similarly, the boundary condition of D1-branes in the free field theory with the electric flux f, which we can obtain from eq. (3.34) by taking the T-duality of a D0-brane, is given by

$$\partial_1 T - if\partial_2 Y = 0\,, \qquad \partial_1 Y - if\partial_2 T = 0\,, \qquad \partial_2\varphi'' = 0\,. \tag{3.42}$$

In the same method as before we can determine its interpretation in the curved spacetime (2.1). The result is a D1-brane wrapped on the spiral string

$$\hat{\varphi} + \frac{f}{2}(\beta - \alpha)y - \frac{f}{2}(\alpha + \beta)t = \text{const}\,, \tag{3.43}$$

with the electric flux $F_{yt} = -f$. This will be understood as the geodesic surface in the spacetime. From (3.43) we can explain the previous condition (3.31) of moving away from the origin if we remember the periodicity $y \sim y + 2\pi R$ and $\varphi \sim \varphi + 2\pi$ as in [14, 15].

3.6 D-brane instanton

In our model the time direction is also curved as well as space directions and thus it will be interesting to examine D-brane instantons,[15] which have the Dirichlet boundary condition in the time direction

$$\partial_\sigma T|_{\tau=0}|B\rangle = 0: \qquad (\beta - \alpha)\hat{J}_0|B\rangle = 0\,. \tag{3.44}$$

The condition (3.44) shows that $\hat{J}$ should be zero unless $\alpha = \beta$ (Melvin model). Then (below we assume $\alpha \neq \beta$) we can construct a D0-brane (and D1-brane) only at the origin and they cannot move. A D2-brane (and D3-brane) can also exist. These results would be useful when we consider the non-perturbative effect by D-instantons; they show that we can neglect the instanton effect of D0 and D1-branes if we concentrate on physics at the large value of ρ.

4. Penrose limit in NSNS background

In this section we would like to discuss the Penrose Limit [18] of superstring backgrounds with NSNS flux and to apply our previous results on D-branes to these limits. The Penrose limit gives us the notion of a lorentzian version of tangent plane. Thus this is a useful tool of approximating a curved spacetime. This limit has recently been applied to the background $AdS_p \times S^q$ with RR-flux and found to be the maximally supersymmetric pp-wave background [19, 34]. This background is exactly solvable in the light-cone Green-Schwarz string theory [19, 45] (see also [33, 46]) and the duality between gauge and gravity theory has been checked even for stringy excitations [19] (for general discussions on the holographic relation see also [25]). Here we mainly concentrate on the background with NSNS flux.

The most simplest example is the Penrose limit of NS5-branes. This was analyzed in [21] and it was found that the limit is equivalent to the product of Nappi-Witten model (NW model) [17] defined by the action (2.20) and the six-dimensional flat space $\mathbb{R}^6$. The half of the maximal supersymmetries are preserved in this model, which is the same number as in the original NS5-branes.

It is interesting to apply this limit to the near horizon limit of F1-N5 system, that is, $AdS_3 \times S^3$ [19, 33]. The result is given by the six-dimensional generalization of NW model. Since we have the half of the maximal supersymmetries in all of these kinds of NW model, this again reproduces the original number of supersymmetries (sixteen) in $AdS_3 \times S^3$.

[15] Recently, D-brane instantons were reinterpreted from the viewpoint of tachyon condensation [44].

JHEP05(2002)012

4.1 Supersymmetry enhancement in the Penrose limit of spacetime coset model

One of the interesting phenomena observed in the discussion on the Penrose limit of RR-flux background is the enhancement of supersymmetry[16] [20, 21, 22]. This phenomenon was found in the example of $AdS_5 \times T^{1,1}$, which preserves 1/4 of the maximal supersymmetries. The manifold $T^{1,1}$ is defined to be a coset $\mathrm{SU}(2)\times\mathrm{SU}(2)/\,\mathrm{U}(1)$. If we take the Penrose limit in this model, it is enhanced maximally [20, 21, 22]. In the appendix C we show that such an enhancement also occurs in another supersymmetric coset system $W_{4,2} \times S^5$ considered in [30], where $W_{4,2}$ is a lorentzian coset of $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SL}(2,\mathbb{R})/\,\mathrm{U}(1)$. On the other hand, if we consider orbifolds of the pp-waves, then the supersymmetry is generically[17] reduced and can be interpreted in terms of the dual quiver gauge theory [20, 23].

Now we are interested in possibilities of supersymmetry enhancement in the background with only NSNS flux. As a particular example we would like to examine the Penrose limit of the model constructed in [29]. The spacetime in this model is described by the coset SCFT

$$\frac{\mathrm{SL}(2,\mathbb{R})_{k_1+2} \times \mathrm{SL}(2,\mathbb{R})_{k_2+2}}{\mathrm{U}(1)} \times \frac{\mathrm{SU}(2)_{k_1-2} \times \mathrm{SU}(2)_{k_2-2}}{U(1)}\,, \tag{4.1}$$

where k_1 and k_2 are the level of the WZW model and we can see the total central charge of the model agrees with the correct value. We denote the first (non-compact) manifold by W and the second by T. It is known that this model is non-supersymmetric [29].

The explicit metric is given by ($k \equiv k_1$ and $Q^2 \equiv k_2/k_1$)

$$\begin{aligned} ds^2 \;=\; & k[(d\theta_1)^2 + \sin^2\theta_1 (d\phi_1)^2] + kQ^2[(d\theta_2)^2 + \sin^2\theta_2(d\phi_2)^2] + \\ & + k(d\psi + \cos\theta_1 d\phi_1 + Q\cos\theta_2 d\phi_2)^2 + \\ & + k[(dr_1)^2 + \sinh^2 r_1 (d\tilde\phi_1)^2] + kQ^2[(dr_2)^2 + \sinh^2 r_2 (d\tilde\phi_2)^2] - \\ & - k(dt + \cosh r_1 d\tilde\phi + Q\cosh r_2 d\tilde\phi_2)^2\,. \end{aligned} \tag{4.2}$$

Then the metric of T (the former part) shows the coset space is $T^{1,Q}$ type even though it is not Einstein metric. It satisfies the equation of motion with the non-trivial B-field

$$B = k\cos\theta_1 d\phi_1 \wedge d\psi + kQ\cos\theta_1\cos\theta_2 d\phi_1 \wedge d\phi_2 - kQ\cos\theta_2 d\phi_2 \wedge d\psi\,. \tag{4.3}$$

In order to discuss its Penrose limit, we would like to take the large volume limit $k \to \infty$. Let us take the following limit

$$\begin{aligned} & x^+ = \frac{1}{2}\Big(t + \tilde\phi_1 + Q\tilde\phi_2 + \psi + \phi_1 + Q\phi_2\Big)\,, \qquad x^- = \frac{k}{2}\Big(t + \tilde\phi_1 + Q\tilde\phi_2 - (\psi + \phi_1 + Q\phi_2)\Big)\,, \\ & \theta_1 = \frac{\xi_1}{\sqrt{k}}\,, \qquad \theta_2 = \frac{\xi_2}{\sqrt{k}Q}\,, \qquad r_1 = \frac{\rho_1}{\sqrt{k}}\,, \qquad r_2 = \frac{\rho_2}{\sqrt{k}Q}\,. \end{aligned} \tag{4.4}$$

[16]For the discussions of supersymmetry of pp-waves in a more general context and its connection with the worldseet supersymmetry of the massive string action see also [26].

[17]Recently, the example of supersymmetry enhancement in orbifolded pp-waves was also interpreted holographically [28].

Finally we obtain the pp-wave metric

$$ds^2 = -4dx^+dx^- - (\xi_1^2 d\phi_1 + \frac{\xi_2^2}{Q}d\phi_2 + \rho_1^2 d\tilde{\phi} + \frac{\rho_2^2}{Q}d\tilde{\phi}_2)dx^+ + + \sum_{i=1}^{2}(d\xi_i^2 + \xi_i^2 d\phi_i^2) + \sum_{i=1}^{2}(d\rho_i^2 + \rho_i^2 d\tilde{\phi}_i^2)\,. \tag{4.5}$$

and the B-field

$$B = -\left(\frac{\xi_1^2}{2}d\phi_1 + \frac{\xi_2^2}{2Q}d\phi_2\right)\wedge dx^+ - \left(\frac{\rho_1^2}{2}d\tilde{\phi}_1 + \frac{\rho_2^2}{2Q}d\tilde{\phi}_2\right)\wedge dx^+ + (\text{trivial part})\,. \tag{4.6}$$

On the other hand, the ten-dimensional NW model[18] is defined by the action

$$S = \frac{1}{\pi\alpha'}\int (d\sigma)^2 \left[\partial u\bar{\partial}v + \sum_{i=1}^{4}(\beta_i\ \rho_i^2\ \partial u\bar{\partial}\phi_i + \partial\rho_i\bar{\partial}\rho_i + \rho_i^2\partial\phi_i\bar{\partial}\phi_i)\right]. \tag{4.7}$$

Then our results (4.5) and (4.6) show that the Penrose limit of (4.1) is equivalent to the generalized NW model (4.7) with the specific parameters $\beta_1 = \beta_3 = -1$, $\beta_2 = \beta_4 = -1/Q$.

The spectrum can be computed as follows

$$E^2 - \vec{p}^2 = \frac{2}{\alpha'}(\hat{N}_L + \hat{N}_R) + p_y^2 - 2(p_y + E)\left(\sum_{i=1}^{4}\beta_i\hat{J}_{iR}\right). \tag{4.8}$$

In this limit we can conclude the half of maximal supersymmetries are enhanced in spite of starting with the non-supersymmetric model. This implies that there are supersymmetric sectors in the whole non-supersymmetric holographic dual theory.

4.2 D-branes in Nappi-Witten background

Next let us apply our previous results of D-branes to the Penrose limit[19] of the model (2.2), namely NW model. Since the supersymmetry of the bulk sector is restored in this limit, here we are interested in the supersymmetry of D-branes (for the analysis of D-branes in NW model from the viewpoint of current algebra see [48]). The NW model, as we have seen in section 2.3, can be obtained by setting $R \to \infty$ and $q_\pm = 0$, $\alpha = 0$. Because in this limit there are no winding modes in the boundary condition (3.2), we must always set $\hat{J} = 0$ for a D0-brane. Thus a D0-brane can only exist at the origin.[20] An important result is that the supersymmetry seems to be broken on this brane as Bose-Fermi degeneracy can not be seen in the vacuum amplitude (A.2). The corresponding open string hamiltonian is given by

$$H_o = -\alpha' E^2 + \frac{\alpha'}{4}\beta^2\hat{J}^2 + \hat{N}\,. \tag{4.9}$$

Note that we can see that there are no tachyons on D-branes. On the other hand, the D1-brane in this limit has the similar property to a D1-brane in flat space. For example, it is easy to see that its vacuum amplitude is the same as in the flat space and thus does vanish. It can also move away from the origin.

[18] For higher-dimensional NW models see [47].

[19] For recent discussions on D-branes in pp-wave (RR) background see [24].

[20] Even if we consider the boosted D0-branes as before in order to move around, we must set $v = -1$ and thus they become singular.

5. Conclusions and discussions

In this paper we investigated the superstring version of the exactly solvable model [1], which describes a curved spacetime with four parameters. In particular, we constructed the boundary states of D-branes in this model and calculated their open string spectra. Generally, in this model supersymmetries are completely broken and closed string tachyons appear. For a particular region of the parameters we have spacetime singularities. However, we show that the open string spectrum does not become singular at high energy for any value of parameters due to α' corrections. This seems to suggest that the phenomenon is due to a stringy resolution of spacetime singularity and we may allow such a singularity. In other words, the free field representation of the background may give a kind of analytic continuation of the singular spacetime. One of the well-studied example of space singularities smoothed in string theory is the orbifold singularity (see e.g. [49]). Our example may be another type of resolution of spacetime singularities. We would like to leave the detailed interpretation of this as a future problem.

Though our analysis of D-branes can be seen as a generalization of the previous analysis on D-branes in Melvin background [14, 15, 16], we found a crucial difference. We show that we can always construct bulk (or movable) D0-branes in the analysis of the free field representation except the parameter region where it is equivalent to the Melvin model [7]. This result is consistent with the geometrical interpretation of a bulk D0-brane as a D2-D0 bound state wrapped on a torus with an electric and magnetic flux. This phenomenon implies that the geometry is smooth as the brane can probe the whole spacetime.

Furthermore, our model includes the Nappi-Witten model as the Penrose limit. We examined D-branes in this model by using the general results. The Nappi-Witten like models can appear in many examples of Penrose limits of backgrounds with NSNS B flux. Especially, we showed the non-supersymmetric spacetime coset model becomes the higher-dimensional Nappi-Witten model and thus supersymmetric. Such an enhancement of supersymmetry can also be seen in the Penrose limit of 1/4 BPS spacetime coset model with RR-flux and this may be a clue to investigating its holographic dual gauge theory.

Acknowledgments

We would like to thank T. Eguchi for valuable advice and encouragements. We are grateful to Y. Hikida, K. Sakai, Y. Sugawara, T. Uesugi and S. Yamaguchi for stimulating discussions. The research of T.T was supported in part by JSPS Research Fellowships for Young Scientists.

A. Fractional branes

Here we show that the (boosted) fractional D0-branes satisfy the Cardy's condition. For detailed convention of boundary states see [15].

Fractional D0-brane. First, let us consider a fractional D0-brane. Its boundary state is

$$|B\rangle = \sum_{n=-\infty}^{\infty} f_\gamma |n\rangle\!\rangle \otimes |B,\gamma\rangle\!\rangle_{NSNS,RR}\,, \tag{A.1}$$

where the total boundary state $|B\rangle$ consists of the summation of boundary state $|n\rangle\!\rangle \otimes |B,\gamma\rangle\!\rangle_{NSNS,RR}$ with fixed momentum n in the $\mathbb{S}^1$ direction. Its vacuum amplitude is given by

$$\begin{aligned}\mathcal{A} = & \frac{\alpha' V_0}{8\pi R}\sum_{\gamma\in\mathbb{Z}} |f_\gamma|^2 \int ds\,(2\pi\alpha' s)^{-4} \exp\left(-\frac{s\alpha' n^2}{2R^2}\right)\eta(\tau)^{-12}\times \\ & \times\left[\theta_3(0|\tau)^4 - (-1)^\gamma\theta_4(0|\tau)^4 - \theta_2(0|\tau)^4\right] + \\ & + \frac{i\alpha' V_0}{8\pi R}\sum_{\gamma\notin\mathbb{Z}}(-1)^{[\gamma]}|f_\gamma|^2\int ds\,(2\pi\alpha' s)^{-3}\exp\left(-\frac{s\alpha' n^2}{2R^2}\right)\eta(\tau)^{-9}\theta_1(\nu|\tau)^{-1}\times \\ & \times\left[\theta_3(0|\tau)^3\theta_3(\nu|\tau) - \theta_4(0|\tau)^3\theta_4(\nu|\tau) - \theta_2(0|\tau)^3\theta_2(\nu|\tau)\right],\end{aligned} \tag{A.2}$$

where $\tau = is/\pi$, $\nu = \gamma\tau$.

On the other hand, the open string 1-loop amplitude is

$$Z_o = 2\times\int_0^\infty \frac{dt}{2t}\,\mathrm{Tr}_{NS-R}\left[\frac{1+(-1)^F}{2}q^{H_o}\right], \qquad q \equiv e^{-2\pi t}\,, \tag{A.3}$$

where we have defined $\mathrm{Tr}_{NS-R} = \mathrm{Tr}_{NS} - \mathrm{Tr}_R$; the operator H_o denotes the open string hamiltonian

$$H_o = -\alpha' E^2 + \alpha'\left(\frac{Rw}{\alpha'} - \frac{\alpha+\beta}{2}\hat{J}\right)^2 + \hat{N}\,. \tag{A.4}$$

By requiring the equality between eq. (A.2) and (A.3), we obtain

$$\gamma\in\mathbb{Z}: \qquad f_\gamma = \frac{T_0}{2}\,, \qquad \gamma\notin\mathbb{Z}: \qquad f_\gamma = \frac{1}{\sqrt{2}}\left(\frac{|\sin\pi\gamma|}{2\pi^2\alpha'}\right)^{1/2} T_0\,, \tag{A.5}$$

where we have defined $T_0 = \sqrt{\pi}(2\pi\sqrt{\alpha'})^3$.

Bulk D0-brane at $\rho = 0$. If the parameter $\frac{(\alpha+\beta)}{2R}\alpha'$ is rational ($\equiv k/N$), there is a bulk D0-brane. At first, by using U(1) phase in the coefficient f_γ, we can write a fractional brane at $Y = y_0$ as

$$\begin{aligned}|B, y_0\rangle = & \frac{1}{2\pi R}\sum_n f_\gamma e^{i\frac{n}{R}(\hat{y}-y_0)}\Big[|\gamma,+\rangle_{NS} - (-1)^{[\gamma]}|\gamma,-\rangle_{NS} + \\ & + |\gamma,+\rangle_R + (-1)^{[\gamma]}|\gamma,-\rangle_R\Big]\,.\end{aligned} \tag{A.6}$$

If we set N fractional D0-branes on the condition

$$|B\rangle_{\mathrm{bulk}} \equiv \sum_{a=0}^{N-1}|B, y_a\rangle\,, \qquad y_a \equiv \tilde{y}_0 + \frac{2\pi R}{N}a\,, \qquad 0 \le \tilde{y}_0 < \frac{2\pi R}{N}\,, \tag{A.7}$$

only the sectors of $n = N\ell\,(\gamma = k\ell \in \mathbb{Z})$ survive, then $|B\rangle_{\rm bulk}$ describes a bulk D0-brane at $\rho = 0$. If we set $\tilde{y}_0 = 0$ for simplicity, the explicit form of $|B\rangle_{\rm bulk}$ is

$$
\begin{aligned}
|B\rangle_{\rm bulk} = \frac{N}{2(2\pi R)} \sum_{\ell} \frac{T_0}{2} e^{i\frac{N}{R}\ell\hat{y}} \Big[&|0,+\rangle_{NS} - (-1)^{k\ell}|0,-\rangle_{NS} + \\
&+ |0,+\rangle_R + (-1)^{k\ell}|0,-\rangle_R \Big] .
\end{aligned}
\tag{A.8}
$$

The amplitude between two $|B\rangle_{\rm bulk}$ is

$$
\begin{aligned}
\mathcal{A}_{\rm bulk} = \frac{\alpha' V_0}{8\pi R} \sum_{\ell=-\infty}^{\infty} \left(\frac{NT_0}{2}\right)^2 \int ds\, (2\pi\alpha' s)^{-4} \exp\left(-\frac{s\alpha'(N\ell)^2}{2R^2}\right) \eta(\tau)^{-12} \times \\
\times \left[\theta_3(0|\tau)^4 - (-1)^{k\ell}\theta_4(0|\tau)^4 - \theta_2(0|\tau)^4\right] .
\end{aligned}
\tag{A.9}
$$

Therefore by performing the modular transformation of eq. (A.9), we obtain

$$
Z_o = 2N \times \int_0^\infty \frac{dt}{2t}\, {\rm Tr}_{NS-R}\left[\frac{1+(-1)^F}{2} q^{H_o}\right] , \tag{A.10}
$$

where H_o is

$$
H_o = -\alpha' E^2 + \frac{R^2}{\alpha' N^2}(w - k\hat{J})^2 + \hat{N} . \tag{A.11}
$$

Boosted fractional D0-brane. Then we consider the boosted fractional D0-branes. First, we must add the Lorentz contraction factor $\sqrt{1-v^2}$ to the vacuum amplitude $\mathcal{A}$. This factor is absorbed when we perform the Poisson resummation formula with respect to n

$$
\begin{aligned}
\sum_n \sqrt{(1-v^2)} \exp\left[-\frac{\alpha'(1-v^2)}{2R^2 t} n^2 + 2\pi i \gamma_{v0} n \hat{J}\right] = \\
= \sqrt{\frac{2R^2 t}{\alpha'}} \sum_w \exp\left[-\frac{2\pi R^2 t}{\alpha'(1-v^2)}(w - \gamma_{v0}\hat{J})^2\right] .
\end{aligned}
$$

Then we find that the boosted fractional D0-branes satisfy the Cardy's condition and the open string hamiltonian is

$$
H_o = -\alpha' E^2 + \frac{R^2}{\alpha'(1-v^2)}(w - \gamma_{v0}\hat{J})^2 + \hat{N} . \tag{A.12}
$$

Boosted bulk D0-brane at $\rho = 0$. Finally, when $\gamma_{v0} = k/N$, the boosted bulk D0-brane at $\rho = 0$ consists of N boosted fractional D0-branes whose positions are $Y = y_0, y_0 + 2\pi R/N, \ldots$. Its open string hamiltonian is given by

$$
H_o = -\alpha' E^2 + \frac{R^2}{\alpha' N^2 (1-v^2)}(w - k\hat{J})^2 + \hat{N} . \tag{A.13}
$$

The multiplicity of the open string channel is the same as before.

B. Bulk D0-brane in terms of Bessel functions

Here we show the detailed calculations of boundary state with $\rho \neq 0$ by using the Bessel functions. Even though the analysis in section 3.2 can be done without the Bessel function representation, the geometrical interpretation will be more transparent by using them. In particular, this would be helpful if one computes the couplings of D-branes with closed string.

The only non-trivial part is the bosonic zero modes (x_1, x_2) (or (k_1, k_2)) of two-dimensional free fields (X_1, X_2), which represent two-dimensional plane. It is equivalent to quantum mechanics of a free particle and below we will use the polar coordinates (ρ, φ). The hamiltonian is

$$2H = -\frac{\alpha'}{2}\left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}\right) = -\frac{\alpha'}{2}\left[\frac{1}{\rho^2}\frac{\partial^2}{\partial \varphi^2} + \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial}{\partial \rho}\right)\right]. \tag{B.1}$$

Its eigen functions and eigen values are[21]

$$\psi(\rho, \varphi) = J_m(k\rho)\, e^{im\varphi}\,, \qquad 2H\psi(\rho,\varphi) = \frac{\alpha'}{2}k^2\psi(\rho,\varphi)\,, \tag{B.2}$$

where k is the magnitude of the momentum which takes non-negative value and m is the angular momentum which takes integer $(= \hat{J})$ values. $J_m(z)$ is Bessel function defined as

$$J_m(z) \equiv \left(\frac{z}{2}\right)^m \sum_{n=0}^{\infty} \frac{(-1)^n (z/2)^{2n}}{n!(m+n)!}\,, \qquad J_{-m}(z) \equiv (-1)^m J_m(z) \qquad \text{for } m \geq 0\,. \tag{B.3}$$

The generating function of Bessel functions is given by

$$e^{iz\sin\theta} = \sum_{m=-\infty}^{\infty} J_m(z) e^{im\theta}\,. \tag{B.4}$$

Then we can expand a plane wave in terms of Bessel functions

$$e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x_0})} = \sum_{m,n=-\infty}^{\infty} J_m(k\rho) J_n(k\rho_0)(-1)^n e^{im\varphi} e^{in\varphi_0} e^{i(m+n)\theta}\,, \tag{B.5}$$

where

$$\mathbf{x} \equiv (\rho\cos\varphi, \rho\sin\varphi)\,, \qquad \mathbf{x_0} \equiv (\rho_0\cos\varphi_0, \rho_0\sin\varphi_0)\,, \qquad \mathbf{k} \equiv (k\cos\theta, k\sin\theta)\,. \tag{B.6}$$

By using eq. (B.5), we obtain the expansion of a delta function

$$\delta^2(\mathbf{x} - \mathbf{x_0}) = \frac{1}{2\pi}\sum_{m=-\infty}^{\infty}\int_0^{\infty} dk\, k J_m(k\rho) J_m(k\rho_0) e^{im(\varphi - \varphi_0)}\,. \tag{B.7}$$

[21] Here we impose the the smooth boundary condition at the origin $\psi(0,\varphi)$ =finite as usual and thus we do not consider the independent solutions of $N_m(k\rho)$ (Neumann function).

Finally, let us consider the superposition of N delta functions at

$$\mathbf{x}_{0a} = \left(\rho_0, \varphi_0 + \frac{2\pi}{N}a\right), \qquad a = 0, \ldots, N-1\,. \tag{B.8}$$

Then, the wave function of a bulk D0-brane at $\rho_0 \neq 0$ is written as follows

$$\psi_{\text{bulk}}(\mathbf{x}) = \frac{1}{N}\sum_{a=0}^{N-1}\delta^2(\mathbf{x}-\mathbf{x}_{0a}) = \frac{1}{2\pi}\sum_{j=-\infty}^{\infty}\int_0^\infty dk\, kJ_{Nj}(k\rho)J_{Nj}(k\rho_0)e^{-iNj(\varphi-\varphi_0)}\,. \tag{B.9}$$

Notice that only $m \equiv 0 \pmod N$ states survive in eq. (B.9). This is consistent with the boundary condition (3.7). Also if we set $\rho_0 = 0$, only $m = 0$ states survive because $J_m(0) = 0$ for $m \neq 0$. This is also consistent with the fact that $\hat{J}_0 = 0$ when a D0-brane is at the origin.

Vacuum amplitude. Next we calculate the vacuum amplitude of the bulk D0-brane at ρ_0

$$\begin{aligned}\mathcal{A}_X &\equiv \int d^2x\, \psi^*_{\text{bulk}}(\mathbf{x})e^{-2(H+\Delta H)s}\psi_{\text{bulk}}(\mathbf{x}) \\ &= \frac{1}{2\pi}\int_0^\infty dk\, k\sum_{j=-\infty}^{\infty}J^2_{Nj}(k\rho_0)e^{-\frac{\alpha'}{2}k^2 s}e^{\frac{(\alpha-\beta)^2N^2}{8}\alpha' sj^2}\,,\end{aligned} \tag{B.10}$$

where $2\Delta H = -\frac{(\alpha-\beta)^2}{8}\alpha'\hat{J}_0^2$. This is equivalent to the calculation in the section 3.2 with $\tilde{q} = 0$. By using the formula

$$J_n^2(z) = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\theta\, e^{2in\theta}\, J_0(2z\sin\theta)\,, \tag{B.11}$$

we can rewrite (B.10) as

$$\begin{aligned}\mathcal{A}_X = \frac{1}{(2\pi)^2}&\int_0^\infty dk\, ke^{-\frac{\alpha'}{2}k^2 s}\int_{-\pi}^{\pi} d\theta\, J_0(2k\rho_0\sin\theta)\times \\ &\times\sum_{j=-\infty}^{\infty}\exp\left[\frac{\pi(\alpha-\beta)^2\alpha' N^2}{8t}j^2 + 2\pi i\frac{\theta N}{\pi}j\right].\end{aligned} \tag{B.12}$$

Next, after performing the Poisson resummation with respect to j, we integrate out k by using the formula

$$\int_0^\infty dx\, xe^{-a^2x^2}J_0(bx) = \frac{e^{-b^2/4a^2}}{2a^2}\,. \tag{B.13}$$

Then eq. (B.12) changes to

$$\begin{aligned}\mathcal{A}_X = \frac{1}{2\pi N}\frac{i\sqrt{2\alpha' t}}{(\alpha-\beta)\alpha'\pi}&\sum_{m=-\infty}^{\infty}\int_{-\pi}^{\pi} d\theta\, \frac{\exp\left[-\frac{2\rho_0^2\sin^2\theta}{\alpha' s}\right]}{\alpha' s}\times \\ &\times\exp\left[\frac{8\pi t}{(\alpha-\beta)^2\alpha'}\left(\frac{\theta}{\pi}-\frac{m}{N}\right)^2\right].\end{aligned} \tag{B.14}$$

Since we can rewrite the integral of θ by employing

$$\sum_{m=-\infty}^{\infty}\int_{-\pi}^{\pi} d\theta\, f(\sin^2\theta)g\left(\frac{\theta}{\pi}-\frac{m}{N}\right) = 2\sum_{m=0}^{N-1}\int_{-\infty}^{\infty} d\theta\, f\left[\sin^2\left(\theta+\frac{m}{N}\pi\right)\right] g\left(\frac{\theta}{\pi}\right), \tag{B.15}$$

we obtain

$$\begin{aligned}\mathcal{A}_X &= \frac{2}{2\pi N}\frac{i\sqrt{2\alpha' t}}{(\alpha-\beta)\alpha'^2 s\pi}\times \\ &\quad\times\sum_{m=0}^{N-1}\int_{-\infty}^{\infty} d\theta\,\exp\left[-\frac{2\rho_0^2\sin^2(\theta+\pi m/N)}{\alpha' s}+\frac{8\pi t}{(\alpha-\beta)^2\alpha'}\left(\frac{\theta}{\pi}\right)^2\right].\end{aligned} \tag{B.16}$$

Finally, we can rewrite eq. (B.16) by setting $E = 2\theta/(\alpha-\beta)\alpha'\pi$

$$\begin{aligned}\mathcal{A}_X &= \frac{i\sqrt{2\alpha' t}}{2\pi\alpha' s}\frac{1}{N}\sum_{m=0}^{N-1}\int_{-\infty}^{\infty} dE e^{-2\pi t H_o} \\ H_o &= -\alpha' E^2 + \frac{\rho_0^2}{\pi^2\alpha'}\sin^2\left[\frac{(\alpha-\beta)}{2}\pi\alpha' E + \frac{\pi m}{N}\right].\end{aligned} \tag{B.17}$$

These results are consistent with eq. (3.17).

At last, if we take the limit $\rho_0 \to 0$ in eq. (B.17), $\mathcal{A}_X$ approaches to $1/2\pi\alpha' s$ which is the value at $\rho_0 = 0$. This means that the limit $\rho \to 0$ is smooth.

C. Penrose limit of spacetime coset with RR-flux

Here we examine the Penrose limit of the spacetime coset model $W_{4,2}\times S^5$, where $W_{4,2}$ is defined to be $\mathrm{SL}(2,\mathbb{R})\times\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$. This background was investigated in the context of AdS/CFT correspondence in [30]. There are eight supersymmetries in this spacetime and the explicit metric is given by

$$\begin{aligned}(ds)^2 &= -\frac{R^2}{9}(dt+\cosh y_1 d\varphi_1+\cosh y_2 d\varphi_2)^2+\frac{R^2}{6}\left(dy_1^2+\sinh^2 y_1 d\varphi_1^2+\sinh^2 y_2 d\varphi_2^2\right)+ \\ &\quad + R^2(d\psi^2\cos^2\theta + d\theta^2+\sin^2\theta d\Omega_3^2)\,.\end{aligned} \tag{C.1}$$

We take the Penrose limit

$$\begin{aligned}x^+ &= \frac{1}{2}\left(\frac{t+\varphi_1+\varphi_2}{3}+\psi\right), \qquad x^- = \frac{R^2}{2}\left(\frac{t+\varphi_1+\varphi_2}{3}-\psi\right), \qquad \rho = \frac{\theta}{R}, \\ y_i' &= \frac{\sqrt{6}y_i}{R}, \qquad \varphi' = \varphi - x^+\,,\end{aligned} \tag{C.2}$$

and finally we obtain the maximally supersymmetric pp-wave background

$$ds^2 = -4dx^+dx^- - (\rho^2+y_1^2+y_2^2)dx^{+2}+y_1^2 d\varphi_1^2 + y_2^2 d\varphi_2^2\,, \tag{C.3}$$

with the corresponding RR-flux. In this way we have observed that the 1/4 supersymmetric background $W_{4,2}\times S^5$ will have enhanced supersymmetry in the Penrose limit. Interestingly, the final result turns out to be equivalent to the Penrose limit of 1/4 supersymmetric background $AdS_5\times T^{1,1}$ discussed in [20, 21, 22]. We hope this result will be helpful for the analysis of holography in such a spacetime coset space.

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